ALL SAINTS' COLLEGE

Mathematics Specialist
Test 6 2016

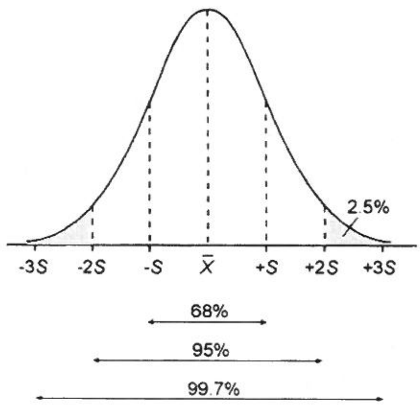
Statistical Inference

NAME: SOLUTIONS

TEACHER: MLA

45 marks
45 minutes

One unfolded A4 page of notes, SCSA formulae booklet and ClassPad calculator permitted



Question 1 [3, 3 & 3 = 9 marks]

(a) What is the mean and standard deviation of a sampling distribution for a binomial distribution with n = 36 and p = 0.7 for samples of size 40?

$\mu_{\bar{x}} = 36 \times 0.7$ $\quad \sigma_{\bar{x}} = \sqrt{\dfrac{36 \times 0.7 \times 0.3}{40}}$ ✓

$= 25.2$ ✓

$= 0.4347$ ✓

(b) The mean time between delivery destinations for a truck driver is 27 minutes, with a standard deviation of 7 minutes.

(i) What is the probability that the next 5 stops will take more than two hours?

$\bar{X} \sim N\left(27, \left(\frac{7}{\sqrt{5}}\right)^2\right)$

$P\left(\bar{X} > \frac{120}{5}\right) = P(\bar{X} > 24)$ ✓ $= 0.8310$ ✓

(ii) Comment on the validity of your answer.

n too small to be confident sampling distribution ✓
is approximately normal.

(c) A company is asked to produce stainless steel rods with lengths accurate to within 0.005mm with a 95% confidence level. If σ = 0.008mm, how many stainless steel rods are necessary to satisfy this request?

$n \geq \left(\dfrac{1.960\,(0.008)}{0.005}\right)^2$ ✓

$n \geq 9.83\ldots$ ✓

So, n must be at least 10 ✓

Question 2 [4 & 1 = 5 marks]

A sample of kookaburras from the Central West region of Western Australia were captured and carefully weighed, yielding the following results (in grams):

460	205	297	494
238	465	299	260
477	344	240	211
419	470	248	271
357	492	362	201

(a) Assuming the sample to be random with a distribution close to normal, determine a 99% confidence interval for the mean weight of kookaburras in Western Australia.

$\bar{x} = 340.5$ ✓

$s_x = 106.9395$ ✓

99% CI for $\mu_x = 2.5758\left(\frac{106.9395}{\sqrt{20}}\right)$

$[278.9, 402.1]$ // grams

(b) Is the confidence level found above valid for the population of kookaburras in Western Australia? Explain.

No. Sample taken from one region ∴ not a random sample. ✓

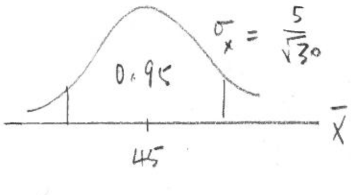
Question 3 [3 & 3 = 6 marks]

The wing-span of a species of bird is known to be normally distributed with length 45cm and variance 25cm. It is suspected that a random sample of 30 birds with mean wing-span of 47cm belongs to the same species.

(a) Determine if the mean length of this sample is significantly different at the 5% level

95% CI for μ_x

$= [43.2, 46.8]$ cm //

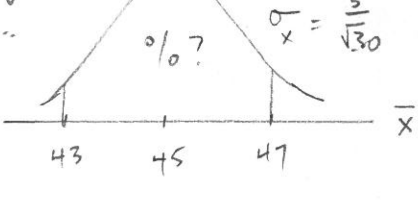


$\bar{X} \sim N\left(45, \left(\frac{5}{\sqrt{30}}\right)^2\right)$

∴ 47cm is significantly different ✓ from the true mean at the 5% level.

(b) Determine the level of significance for the difference between the mean length of this sample and the known length

$P(43 \le \bar{X} \le 47) = 0.9715403...$ ✓



∴ Level of significance $= 2.85\%$ //

Question 4 [1, 1, 2, 2 & 2 = 8 marks]

Medication Alpha is designed to help obese individuals lose weight.

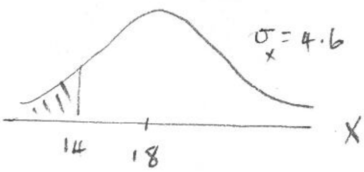
For those prescribed this medication, it was found that weight loss was normally distributed with a mean of 18kg and a standard deviation of 4.6kg.

Let $X \equiv$ weight loss

$X \sim N(18, 4.6^2)$

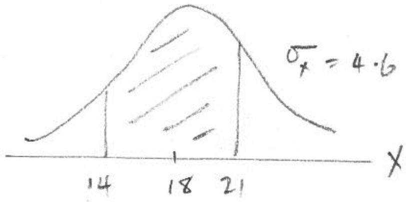
(a) Find the probability that the weight loss was less than 14kg

$P(X < 14) = 0.1923$ ✓



(b) Find the probability that the weight loss was between 14kg and 21kg

$P(14 \leq X \leq 21) = 0.5506$ ✓



(c) If the weight loss was less than 10kg, Medication Beta was administered instead. If 5000 individuals use the initial medication, how many will need to switch to Medication Beta?

$P(X < 10) = 0.0410059\ldots$ ✓

$5000 \times 0.0410059\ldots = 205.0295\ldots$ ✓

$\therefore$ 205 people will switch to Medication Beta

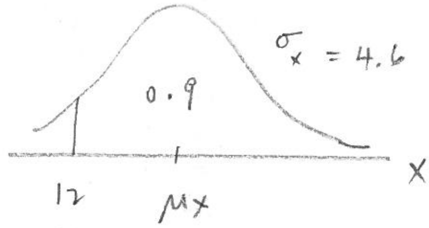
If the physicians require 90% of all individuals to have a weight loss greater than 12kg:

(d) Determine the new mean weight that would correspond to the current standard deviation

Standard score: $z = -1.281552$ ✓

$$-1.281552 = \frac{12 - \mu_X}{4.6}$$

$\therefore \mu_X = 17.9$ kg ✓



(e) If the original mean is maintained, determine the new standard deviation

$$-1.281552 = \frac{12 - 18}{\sigma_x}$$

$\therefore \sigma_x = 4.6818$ kg //

Question 5 [1, 2, 1, 2 & 2 = 8 marks]

An eight-sided die (with faces numbered 1 to 8) is rolled 40 times.

Let $X \equiv$ the number obtained on one roll

Let $\bar{X} \equiv$ the mean of 40 numbers rolled

(a) State the probability distribution for X

$P(X=x) = \frac{1}{8} \; ; \; \{x \in Z : x = 1, 2, 3, 4, 5, 6, 7, 8\}$ ✓

(b) Calculate the mean and standard deviation for X

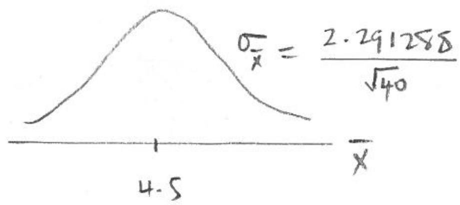
$\mu_X = 4.5$ ✓

$\sigma_X = 2.291288$ ✓

(c) State the probability distribution for $\bar{X}$

$\bar{X} \sim N\left(4.5, \left(\frac{2.291288}{\sqrt{40}}\right)^2\right)$ ✓

ie. $\bar{X} \sim N(4.5, 0.13125)$



(d) Find the probability that for any randomly chosen sample of 40 rolls of the die, the mean of the sample is between 2 and 5

$P(2 \le \bar{X} \le 5) = 0.9162$ ✓✓

(e) Find the probability that in 20 sets of 40 rolls of the die, at least 19 sets would be have sample means between 2 and 5

$X \sim Bin(20, 0.9162)$ ✓

ClassPad : Binomial CD : P (at least 19 sets) = 0.4915 ✓

Question 6 [1, 2, 2, 2 & 2 = 9 marks]

The weights of rainbow trout in Lake Rotorua on New Zealand's North Island are normally distributed with a mean of 2.2kg and a standard deviation of 0.35kg.

(a) What is the likelihood of catching a rainbow trout weighing in excess of 3.0kg?

W ≡ weight of trout

$W \sim N(2.2, 0.35^2)$

$P(W > 3.0) = 0.0111$ ✓

(b) If a rainbow trout is selected at random, what is the chance it weighs less than 1.8kg, given it weighs less than 2.3kg?

$$P(W < 1.8 \mid W < 2.3) = \frac{P(W<1.8)}{P(W<2.3)} = \frac{0.126549}{0.6124515}$$ ✓

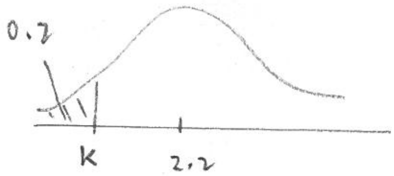
$$= 0.2066$$ ✓

(c) If 20% of the rainbow trout in the lake are underweight, determine the actual weight of an underweight rainbow trout

Let k be weight of an underweight trout

$P(X < k) = 0.2$

$\therefore k = 1.905$ kg ✓✓



(d) Determine the probability that the mean of 25 rainbow trout caught in Lake Rotorua lies between 1.9kg and 2.1kg

$\bar{x} \sim N\left(2.2, \left(\frac{0.35}{5}\right)^2\right)$ ✓

$P(1.9 \leq \bar{x} \leq 2.1) = 0.0766$ ✓

(e) What size sample is required if we wish to be 98% confident the sample mean will not differ from the true mean by more than 200g?

$$n \geq \left(\frac{2.3263479 \times 0.35}{0.2}\right)^2$$

$n \geq 16.57\ldots$ ✓

So, n must be at least 17 ✓